Developer's Tip

An Introduction to the Theory of Planar Failure

In this article we explain the basic concepts behind planar failure analysis of rock slopes. We also discuss the geometric conditions that must be present in order for planar rock slope failure to occur, and we briefly examine the underlying assumptions for such failure modes.

The article will outline a new approach that Rocscience created and used to develop RocPlane, a planar failure analysis program. This approach, founded on elementary vector algebra, generalizes the original method introduced by Hoek and Bray [1]. It can accommodate geometries that are more arbitrary than the ones considered by Hoek and Bray.

Concepts of Planar Rock Slope Failure

Planar rock slope failure occurs when a mass of rock in a slope slides down and along a relatively planar failure surface. The failure surfaces are usually structural discontinuities such as bedding planes, faults, joints or the interface between bedrock and an overlying layer of weathered rock.

Compared to other failure modes encountered in rock slopes, planar failure is relatively rare. Planar failure seldom occurs because the geometric conditions required to produce such failures do not often arise in real slopes. Despite its relative rarity, the study of planar failure mechanisms provides insightful knowledge on the behaviour of rock slopes, and is particularly valuable for investigating the sensitivity of slope behaviour to variations in parameters such as the shear strength of failure surfaces and groundwater conditions.

Conditions for planar failure

There are a number of conditions that must exist in a rock slope for planar failure to occur. First, the failure plane must be sub-parallel to the slope face. This means that it must strike within 20 degrees to the slope face. The failure plane must also daylight into the slope face: it must have a dip that is shallower than the dip of the slope face, and intersect the face above the toe of the slope.

Planar failure also requires the presence of release surfaces (also known in some literature as side relief planes) at the lateral boundaries of the sliding block. These release surfaces must provide insignificant resistance to sliding.

Assumptions in Planar Failure Analysis

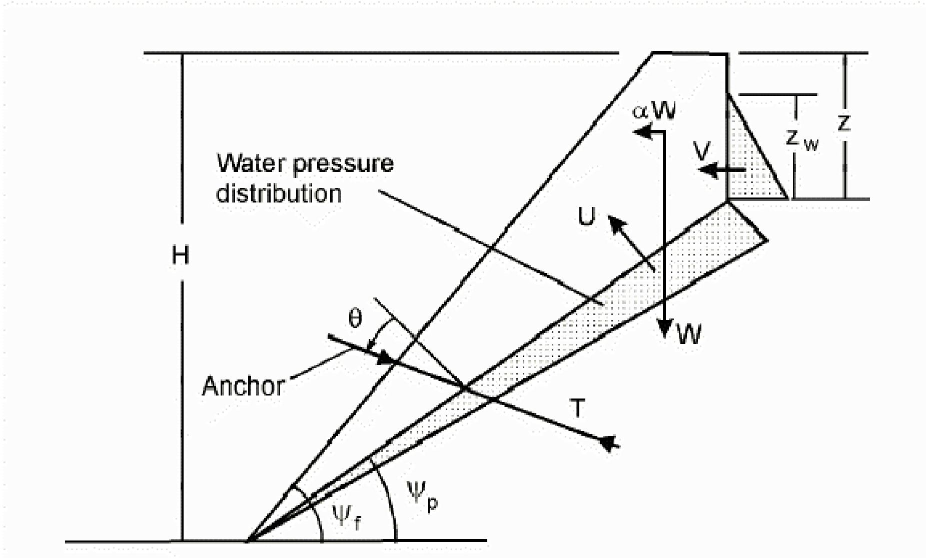
In planar failure mode the sliding mass is assumed to translate as a rigid body down the sliding surface. It does not undergo any rigid body rotation. As such all forces acting on the sliding block are assumed to pass through its centroid. Planar failure analysis considers the sliding block to have unit thickness. This reduces the problem to analysis in two-dimensions only.

As in all limit equilibrium methods, it is assumed in planar failure analysis that all points along the sliding plane are on the verge of failure. It is further assumed that the distribution of stress along the sliding surface is constant, i.e. the strength characteristics along the entire length of the failure surface are all equal.

Combined, the above assumptions represent a rock block sliding along a single joint in a homogeneous rock mass. The failure mechanism and analysis method are therefore very similar to the classical physics case of a block sliding down an inclined plane.

Limitations of Original Planar Rock Slope Analysis Formulations

Original formulations of planar failure analysis of rock slopes all had two fundamental limitations. They involved limit equilibrium equations for a sliding block problem that had a vertical tension crack, and that also assumed the upper surface of the slope to be horizontal. The figure below illustrates a sliding block geometry based on these assumptions.



These two assumptions restrict the generality of the original planar analysis methods. It will be shown later in this article that the restrictions can be eliminated through application of elementary vector algebra.

Definition of the Factor of Safety of a Sliding Block

As explained earlier on, at limit equilibrium all points along the sliding plane are assumed to be near failure. The factor of safety is then defined as the ratio of resisting forces to driving forces, i.e.

$$F = \frac{\text{Total Driving Force}}{\text{Total Resisting Force}} \text{ .}$$

Resisting forces are made up of forces such as the shear strength of the failure plane and other stabilizing forces acting on a wedge. The driving forces consist of the down-slope component of the weight of the sliding block, forces such as those generated by seismic acceleration or by water pressures acting on some faces of the block, and external forces on the upper slope surface.

The factor of safety *F* of the sliding block can be also written as

$$F = \frac{\text{Shear Strength of the Failure Plane}}{\text{Shear Force acting on the Failure Plane}} \text{ .}$$

Advantages of Planar Failure Analysis

At the beginning of this section it was mentioned that planar rock slope failure is relatively uncommon. If this is so then it begs to be asked how applicable planar failure methods are to the design and analysis of real slopes. Why not simply use methods such as vertical slice methods, wedge methods or their three-dimensional extensions that explore more sophisticated failure modes?

The true advantage of planar failure analysis over methods such as vertical slice (Bishop, Janbu, Spencer etc.), two-dimensional wedge (Sarma, Generalized Wedge Method), or their three-dimensional extensions lies in its simplicity. Planar analysis calculations can be done by hand using the equations in Hoek and Bray [1] or the equations in chapter 7 of the free set of rock engineering notes by Evert Hoek (can be found at the Rocscience website, www.rocscience.com). Simple Microsoft Excel™ spreadsheets can be written to perform these calculations. Free programs and web calculators also exist for these calculations.

The simplicity of the calculation procedure for planar failure analysis is also advantageous for performing statistical or risk analysis. Through statistical analysis, the uncertainty in the values of input data can be taken into account, and risks of failure estimated. The method's simplicity of computations facilitates the performance of Monte-Carlo and Latin hypercube statistical simulations in very reasonable computational times.

The planar analysis method enjoys additional advantages such as ease of understanding and interpretation of results, and smaller amounts of input data. As such it can aid engineers in quickly assessing potential trade-offs and alternatives.

RocPlane

RocPlane is a commercial implementation of the planar failure analysis method. This program, developed by Rocscience, distinguishes itself from all other free and commercial packages mainly in its ability to perform statistical planar failure analysis using a wide variety of input data combinations, wedge geometries, and strength and water pressure assumptions. RocPlane also allows engineers to study the sensitivity of the stability of slopes to variations in input data values. This feature coupled with a terrific user interface makes the program a must have for anyone doing this type of analysis.

Extension of Hoek and Bray Method to Arbitrary Block Geometries

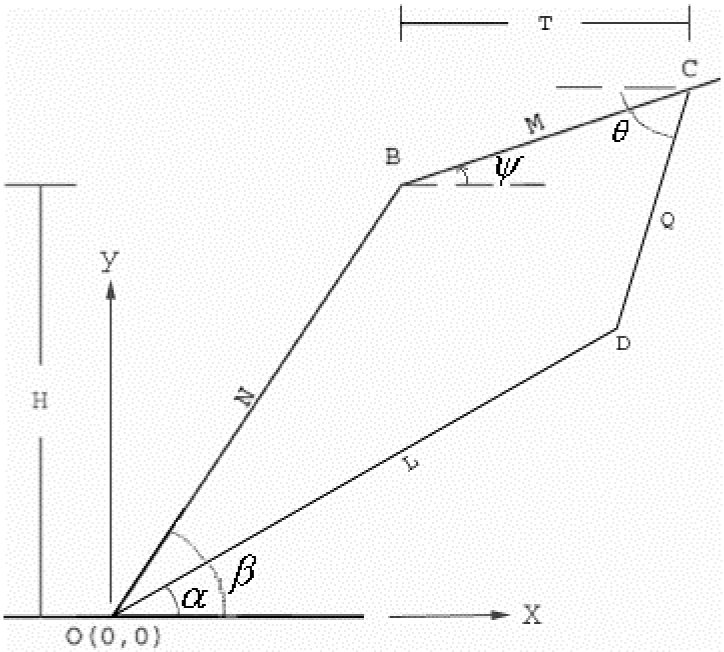
A number of authors such as Sharma et al [2] and Froldi [3] have extended the original planar analysis method to account for non-vertical tension cracks and non-horizontal upper slope surfaces. The problem with most of these extended approaches, however, is the complexity of their formulations. The primary reason for the increased complexity has to do with calculations for determining the area of a sliding block of arbitrary shape.

In this article we would like to present a formulation that equally accounts for any valid wedge geometry but is very simple to understand and implement. The new formulation can accommodate cases of rock slopes involving various combinations of slope face angles, non-vertical tension cracks and non-horizontal upper face angles.

The new method, implemented in RocPlane, computes the area of the sliding block using some simple first-year university vector algebra. Once the area of a wedge or sliding block is computed (and the block's weight since the density of its rock material is known) it follows the same methodology used by the other methods to complete the solution of the problem. The factor of safety is obtained through the application of force equilibrium and failure plane strength principles.

New Vectoral Method for Arbitrary Sliding Block Geometries

In order to derive the equations for the new formulation, we shall first identify the different geometric components that define a sliding block. The figure below depicts the parameters and coordinate axes (*x* and *y*) used to define the problem:



In the analysis the known parameters are:

H – the height of the slope,
b – the slope angle or dip,
a – the failure plane angle or dip,
j – the upper face angle or dip,
q – the tension crack angle or dip,
T – the horizontal distance from the slope crest to the tension crack location on the upper surface,
O – the origin of the Cartesian coordinate system used for the problem. It is situated at the toe of the wedge or sliding mass.

To apply vector algebra techniques to the problem we require the vertices *B*, *C* and *D* of the sliding block. As well we will need to calculate the lengths of the sides of the block, namely *L* (length of the failure plane), *Q* (length of the tension crack), *N* (the slope face length) and *M* (the upper surface length).

Using basic trigonometry, the length *N* and coordinates (B_x, B_y) of the vertex *B* can easily be computed as:

$$N = H / \sin\beta$$

$$B_x = H \cot\beta$$

$$B_y = H \text{ .}$$

Knowing the coordinates of point B, the length T and the angle φ, we can compute the length M and the coordinates of point C using the following relationships:

$$M = T / \cos\varphi$$

$$C_x = B_x + T$$

$$C_y = B_y + T\tan\varphi \ .$$

To obtain the lengths L and Q and the coordinates D_x and D_y of vertex D, we will solve for the intersection of a vector of known orientation θ that starts from the point C, and a vector with known orientation α originating from point O. This solution yields the following results:

$$Q = \frac{C_y \cot\alpha - C_x}{\sin\theta\cot\alpha - \cos\theta}$$

$$L = \frac{C_y - Q\sin\theta}{\sin\alpha}$$

$$D_x = L\cos\alpha$$

$$D_y = L\sin\alpha$$

By obtaining the above vertex coordinates and lengths of the sides of the wedge, we now have all the information required to define the vectors that connect the wedge's vertices. Knowledge of these vectors simplifies computations for the area of the wedge.

The wedge can be split into two triangles ODB and BDC. From linear algebra we know that the area of a triangle is half the cross product of the two vectors originating from any one of the triangle's vertices. Using this formula, the area of the wedge can be computed as

$$A = \frac{1}{2}\left\| B_x D_y - B_y D_x \right\| + \frac{1}{2}\left\| (D_x - B_x)(C_y - B_y) - (D_y - B_y)(C_x - B_x) \right\| \ .$$

A pair of brackets || || in the above formula represent the absolute value of the expression found within the brackets.

Having computed the area of the sliding block, its weight can be determined by multiplying its area with the unit weight, γ, of the rock material of the slope. The weight W is simply

$$W = A\gamma \ .$$

After computing the weight W of the wedge, the method follows that same approach as all others for calculating the factor of safety of the sliding block. This involves the following steps:

Resolution of all forces acting on the sliding block into driving and resisting forces,

Summation of the forces in each class,

Selection of a model or criterion for predicting the shear strength of the failure plane, and

Application of the factor of safety equation.

A Simple Example

For the purposes of illustrating the complete solution of the planar failure problem, we shall consider the simple example of a wedge involving a tension crack that has no forces acting on it other than those due to the self-weight of the wedge. We shall also assume a simple relationship for the shear strength of the sliding surface.

Of the various strength criteria for determining the shear strength of discontinuities, the most commonly used model is probably the Mohr-Coulomb criterion. It is defined as:

$$\tau = c + \sigma_n \tan\phi ,$$

where c is the cohesion, σ_n is the normal stress acting on the failure plane, and ϕ is the friction angle.

Applying the Mohr-Coulomb strength model to the equation, given earlier in the article, that expresses the factor of safety as the ratio of the shear strength of the failure plane to the shear force inducing sliding along the plane, we obtain the result

$$F = \frac{cL + W\cos\alpha\tan\phi}{W\sin\alpha} .$$

Summary

Planar rock slope failure assumes a very simple form of failure that, despite its straightforwardness, offers considerable insights into the behaviour of real slopes. The approach's simplicity endows it with advantages over more sophisticated analysis methods. However, original formulations for performing planar failure analysis had two limitations: they considered only slopes with horizontal upper surfaces and tension cracks that were vertical.

The original formulations were extended to situations that involved non-vertical tension cracks and non-horizontal upper slope faces. These extensions, however, involved more complicated equations as a result of which they lost some of the elegance and simplicity of the method. To restore the ease of the method, we developed a generalized formulation based on simple geometric relationships and vector algebra. The new solution method was used to develop the software RocPlane.

The article has provided an overview of planar failure analysis and the essentials of the new solution method. As an example, the factor of safety equation was derived for the very simple case of a block, which had no external forces acting on it, sliding along a Mohr-Coulomb surface. In more detailed analysis, other forces such as those due to water pressure, seismic loads, external forces, and support forces can be included. For details on how these forces are added to the factor of safety equation you can refer to the RocPlane theory manual.

The article briefly discussed the Mohr-Coulomb strength criterion. However, there are several other strength models in rock engineering, and many of them can be found implemented in RocPlane. In addition to the variety of strength models, RocPlane offers several facilities for performing statistical analysis and sensitivity calculations. The program also includes features for easily building, modifying and viewing models, for analyzing input data and results.

References

1. Hoek, E. and Bray, J.W., “Rock Slope Engineering”, The Institution of Mining and Metallurgy, Third Edition, 1981, pp. 150-198.

2. Sharma, S., Raghuvanshi, T.K., and Anbalagan, R., “Plane failure analysis of rock slopes”, Technical Note, Geotechnical and Geological Engineering, 1995, v.13, pp. 105-111.

3. Froldi, P., “Some Developments to Hoek & Bray’s Formulae for the Assessment of the Stability in case of Plane Failure”, Bulletin of the International Association of Engineering Geology, October 1996, v.54, pp. 91-95.